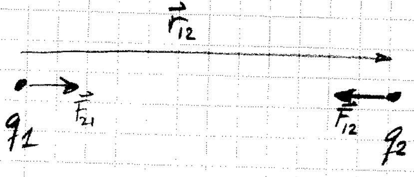
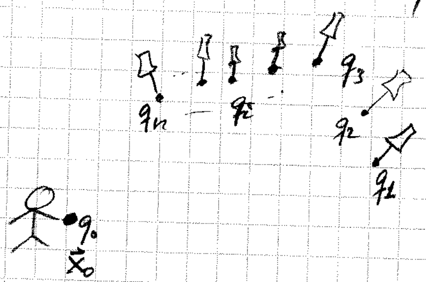
Electric Field

* We know: Coulomb law

$$\vec{F}_{12} = k \cdot \frac{q_1 \cdot q_2}{r_{12}^2} \cdot \hat{r}_{12}$$



* Imagine that we have a (possibly complicated) distribution of charges, glued somewhere so that they can't move.



Say that I'm holding another charge, q_0, and I go close to the other charges.

Q: What force will the charge q_0 feel?

→ superposition principle : $\vec{F}_{on\, q_0} = \vec{F}_{1,0} + \vec{F}_{2,0} + \ldots + \vec{F}_{n,0}$

What happens with a different q_0, say $q_0' = 2q_0$ (twice the charge)?

→ superposition principle : $\vec{F}'_{on\, q_0} = 2\vec{F}_{1,0} + 2\cdot\vec{F}_{2,0} + \ldots + 2\vec{F}_{n,0}$

forces 2x as large because charge q_0 2x as strong.

So $\vec{F}'_{on\, q_0'} = 2 \cdot \vec{F}_{on\, q_0}$

We realize: the only thing that changes by changing q_0 is that the force is scaled by the same factor. Actually, there is a "unit force", the electric field, that tells you how the force will be by multiplying it by q_0:

$$\vec{F}_{on\, q_0} = q_0 \cdot \vec{E}_{at\, \vec{x}_0}$$

where

$$\vec{E}_{at\,\vec{x}_0} = k \cdot \frac{q_1}{r_{1,0}^2}\hat{r}_{10} + k\frac{q_2}{r_{20}^2}\hat{r}_{20} + \dots + k\frac{q_n}{r_{n0}^2}\hat{r}_{n0}$$

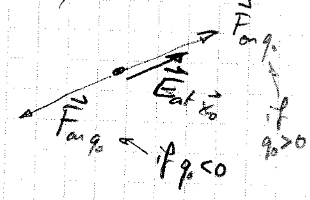
Remarks:

→ $\vec{E}_{at\,\vec{x}_0}$ depends of course on $\vec{x}_0$. At a different point the electric field will be different!

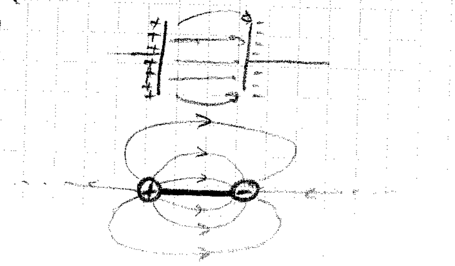
→ $\vec{E}_{at\,\vec{x}_0}$ no longer depends on q_0! It's now completely a property of the original charges: the electric field "was there" before I came along with q_0.

→ $\vec{E}_{at\,\vec{x}_0}$ can be seen as "precalculating the unit force at $\vec{x}_0$ if we don't know q_0 yet", and then the force is simply $\vec{F}_{on\,q_0} = q_0 \vec{E}_{at\,\vec{x}_0}$

→ Works also for negative charges! If q_0 is negative, then the vector flips direction (as expected)



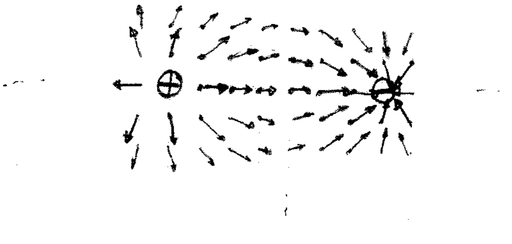
→ If the distribution of charges is very complicated, but we still managed to calculate $\vec{E}_{at\,\vec{x}_0}$ in some way, then the work is done! (And $\vec{F}_{on\,q_0} = q_0 \vec{E}_{at\,x_0}$.) e.g. capacitor plates, dipole, ...



* Field lines.

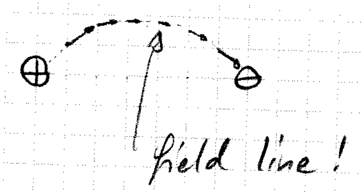
Given that the electric field changes at every point of space, how can we draw it / represent it in a diagram?

→ possible idea: draw the electric field for each point on a grid (for example)

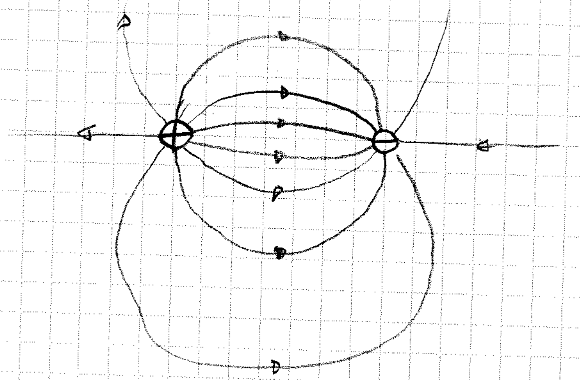


as you can see, it's not handy & cumbesome to draw!

→ other idea: "follow the arrows"



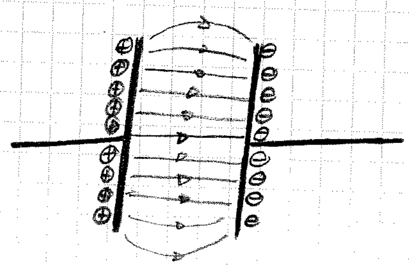
drawing many such field lines represents the electric field at every point in space



* field lines never cross (why? we followed the arrows!)
* density of lines indicates field strength

Further examples:

→ two charged plates (when not close to the edges, the electric field is constant)



→ two positive charges